

Arrow's impossibility theorem, part of "Social choice theory" an economic theory that considers whether a society can be ordered in a way that reflects individual preferences was lauded as major breakthrough. It went on to be widely used for analysing problems in welfare Economics.

Example:-

Let's look at an example illustrating the type of problems highlighted by Arrow's Impossibility Theorem.

Consider that, voters are asked to rank their preferences of candidates A, B and C :-

- * 45 votes $A > B > C$ [45 people prefer A over B and prefer B over C]
- * 40 votes $B > C > A$ [40 people prefer B over C and C over A]
- * 30 votes $C > A > B$ [30 people prefer C over A and A over B]

Candidate (A) has the most votes, so he/she would be the winner. However, if (B) was not running, (C) would be the winner, as more people prefer (C) over (A). (A) would have 45 votes and (C) would have 70. This result is demonstration of Arrow's theorem.

Applicability :-

Arrow's Impossibility Theorem is applicable when voters are asked to rank all candidates. However, there are other popular voting methods, such as approval voting or plurality voting, that don't use this framework.

Explanation

We all know that a Democracy depends on people's voices being heard. For example, when it is time for a new government to be formed, an election is called, and people head to the polls to vote. Millions of voting slips are taken, counted to determine who is the most popular candidate and the next elected official.

According to the Arrow's Impossibility theorem, in all cases where preferences are ranked, it is impossible to formulate a social ordering without violating one of the following conditions:-

- ⇒ Nondictatorship :- The wishes of multiple voters should be taken into consideration.
- ⇒ Pareto Efficiency :- Unanimous individual preferences must be respected. If every voter prefers Candidate (A) over Candidate (B), Candidate (A) should win.
- ⇒ Independence of Irrelevant Alternatives :- If a choice is removed, then the others' order should not change. If Candidate (A) ranks ahead of Candidate (B), Candidate (A) should still be ahead of Candidate (B), even if a third candidate (C), is removed from the participation.
- ⇒ Unrestricted domain :- Voting must account for all individual preferences.
- ⇒ Social ordering :- Each individual should be able to order the choices in any way and indicate ties.

(Contd)....

Good Morning students,

ARROW'S IMPOSSIBILITY THEOREM

History :-

The theorem is named after economist Kenneth J. Arrow. Arrow, who had a long teaching career at Harvard University and Stanford University, introduced the theorem in his doctoral thesis and later popularised it in his book "Social Choice and Individual Values" in 1951. The original paper titled, A Difficulty in the Concept of Social Welfare, earned him the Nobel Memorial Prize in Economic Sciences in 1972.

Arrow's research has also explored the Social choice theory, Endogenous Growth Theory, Collective decision making, the economics of information, and the economics of racial discrimination among other topics.

Introduction:-

Arrow's Impossibility Theorem is a social-choice paradox illustrating the flaws of ranked voting systems. It states that a clear order of preferences cannot be determined while adhering to mandatory principles of fair voting procedures. Arrow's Impossibility Theorem, named after Economist Kenneth John Arrow. It is also known as General Impossibility Theorem.